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Technical Notes

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Oscillatory Behavior of Transonic Aeroelastic Instability Boundaries

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I. Introduction

THE use of computational aeroelasticity employing high-fidelity CFD-based (computational fluid dynamics-based) nonlinear aerodynamics has matured from a research exercise to a powerful tool in engineering applications. The stability of an aeroelastic system can be inferred from a time-marching simulation following an initial excitation. Calculations of complete aircraft configurations have been made [1,2]. The time-accurate approach is very capable due to its generality. However, it carries significant computational costs, in particular to solve for the unsteady, nonlinear transonic aerodynamics. One alternative approach uses the theory of dynamic systems to predict aeroelastic instabilities. An eigenvalue-based calculation solves the stability problem for a steady-state solution of the aeroelastic system instead of performing unsteady simulations [3–6].

The "typical section" aerofoil of Isogai [7,8], used to represent the bending and torsional behavior of a wing structure, is a benchmark case for methods predicting aeroelastic instabilities. Figure 1 shows a comparison between results from different numerical methods [7–11] illustrating the instability boundary as flutter speed index V_F vs freestream Mach number. The s-shape of the curve in the deep transonic region, giving a second stable branch for higher values of the flutter speed index, is distinct for the inviscid aerodynamic modeling approaches. The current solver (BIFOR) allows the instability boundary to be resolved with small steps in Mach number. The average Mach number increment in the "transonic dip" region is 2.0×10^{-3}, giving a total number of more than 200 individual points on the shown boundary, which required less than 4 h of computation on a desktop computer.

Taking a close-up view, the small steps in Mach number reveal an oscillatory behavior in the transonic regime. These oscillations merit a closer investigation, and, therefore, the NACA 0012 aerofoil configuration defined in [3] as "heavy case" is considered in the current note.

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II. Numerical Method

The coupled aeroelastic system of an Euler aerodynamic model and the structural model of a pitch and plunge aerofoil can be written in semidiscrete form as

$$\frac{\mathrm{d}\boldsymbol{w}}{\mathrm{d}t} = \boldsymbol{R}(\boldsymbol{w}, \mu) \tag{1}$$

where $\boldsymbol{w}$ is the vector of the fluid and structural unknowns, and $\boldsymbol{R}$ is the corresponding residual vector. The independent bifurcation parameter μ is chosen to be the reduced velocity $\bar{U}$ as a nondimensional parameter for the freestream velocity. A steady-state solution is obtained by temporal integration of the system in Eq. (1) using a fully implicit time-marching scheme [12]. Spatial discretization of the fluid residual is obtained by Osher's approximate Riemann solver [13]. MUSCL variable extrapolation is used to achieve second-order spatial accuracy, whereas van Albada's limiter is applied to preserve the monotone behavior of a first-order scheme. The applied spatial discretization is a standard approach to produce very sharp shock resolution. The resulting linear system is solved by a preconditioned Krylov subspace method [3,5].

The stability of an aeroelastic system can be judged by examining the temporal response to an initial excitation. Starting from a steady-state solution, a time-accurate unsteady simulation commences with an initial disturbance in the structural solution. Temporal integration of the coupled aeroelastic system is then accomplished by a dual-time stepping method [14], where the coupled system is iterated to a steady state in pseudotime at each real time step.

Alternatively, the stability can be inferred directly from the steady-state solution $\boldsymbol{w}_0$ (referred to as equilibrium) of the coupled aeroelastic system. The theory of dynamic systems gives criteria for an equilibrium to be stable. In particular, stability is determined by the eigenvalues $\lambda_j(\mu)$ of the system Jacobian matrix $A(\boldsymbol{w}_0, \mu)$ evaluated at the equilibrium for varying values of the bifurcation parameter. A stable system has all its eigenvalues with a negative real part. The value of the bifurcation parameter resulting in a complex conjugate pair of eigenvalues with a vanishing real part defines an instability of the Hopf type, which commonly leads to flutter or limit-cycle oscillations. To find the eigenvalues of the Jacobian matrix, the eigenvalue problem

$$(A - \lambda_j I)\boldsymbol{p}_j = 0 \tag{2}$$

is solved for a particular complex eigenpair $(\boldsymbol{p}_j, \lambda_j)$. The matrix A is the exact Jacobian of the spatial scheme [3–5].

III. Results

The transonic instability boundary for the NACA 0012 aerofoil configuration calculated by the eigenvalue-based approach and resolved at about 200 individual Mach numbers is presented in Fig. 2. The critical Mach number of the NACA 0012 aerofoil at 0 mean angle of attack is about 0.727. Exceeding this critical value, and thus developing a significant supersonic region including a shock wave in the near-field region of the aerofoil, leads to the observed oscillatory behavior in the instability curve. A grid refinement study on a three-block C-type family of grids, the results of which are presented in the inlay of Fig. 2, shows a dependence of the oscillatory frequency on the streamwise grid resolution, whereas the normal grid resolution has little influence on the phenomenon. In addition, the strength of

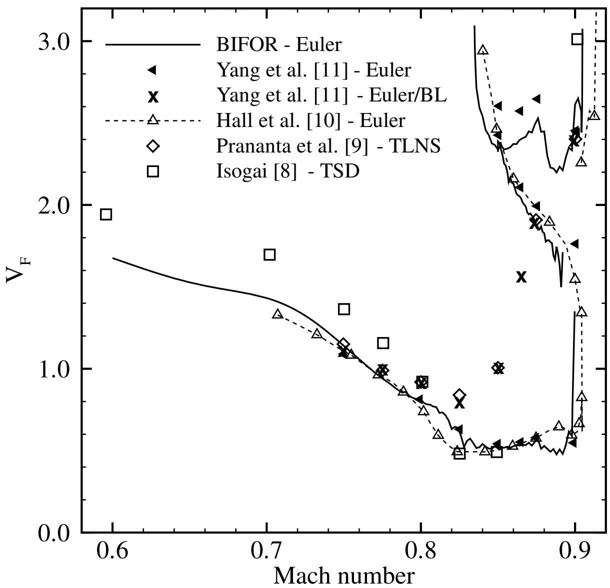

Fig. 1 Isogai case showing the instability boundary as flutter speed index $V_F(=\bar{U}/\sqrt{\mu_s})$ vs Mach number, where BL is the boundary layer, TLNS is thin-layer Navier–Stokesm, and TSD is transonic small disturbance.

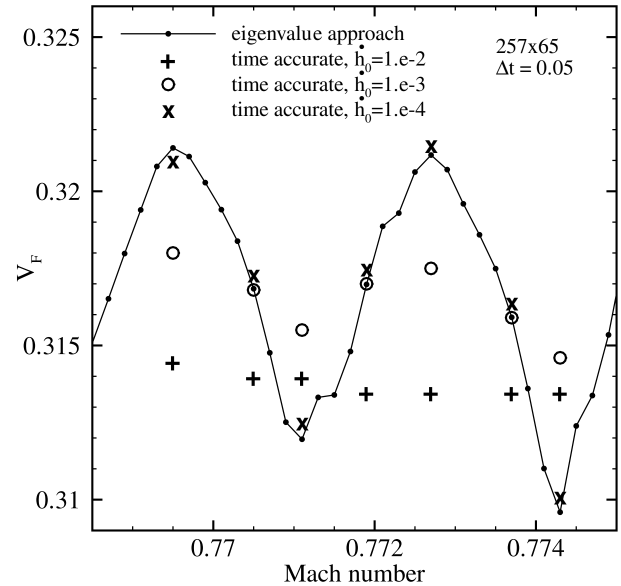

Fig. 3 Crossplot of instability boundaries calculated by eigenvalue-based and time-domain approaches for NACA 0012 aerofoil configuration.

the shock wave affects the oscillatory amplitude, which can be seen in Fig. 2 for the fine grid (257×65 nodes) with increasing freestream Mach number. This observation was confirmed by simulating the instability boundaries for thinner and thicker aerofoils of the NACA four-digit series.

To support the eigenvalue-based results, unsteady simulations were done on the fine grid using a nondimensional time step of 0.05, which is equivalent to about 500 steps per cycle. Time step refinement did not show an influence on the oscillatory effect. Starting from steady-state solutions, time responses for several values of the initial disturbance in the plunge rate were simulated, the results of which are crossplotted with results of the eigenvalue-based approach in Fig. 3. Generally, a dependence of the simulations on the strength of the disturbance can be observed. Small initial deflections give results that are very similar to the eigenvalue-based approach (consistent with a linear stability analysis) revealing the same phenomenon, whereas higher values eliminate the oscillations. In unsteady simulations the location of the shock wave changes depending on the structural solution. The variation of the pressure distribution is very weak if the system is only disturbed slightly, and, hence, the discrete steady-state shock resolution is a strong factor throughout the unsteady response. For larger initial disturbances on the other hand, the influence of the steady state on the time-accurate simulation is dominated by dynamic effects.

In the current eigenvalue-based simulations, the Mach number increment can be decreased easily by orders of magnitude and, thus, approaches a continuous change, whereas the formed shock wave can not move accordingly along the aerofoil but is restricted to the discrete grid location. The discrete steady-state resolution of the shock wave is reflected, for example, in the integrated fluid forces. This is presented in Fig. 4, showing a crossplot of instability boundary and steady-state lift coefficient at a very small angle of attack for a range of transonic freestream Mach numbers. Figure 4 clearly suggests a correlation between the two curves concerning the oscillations. It is important that for simulations of an inviscid duct

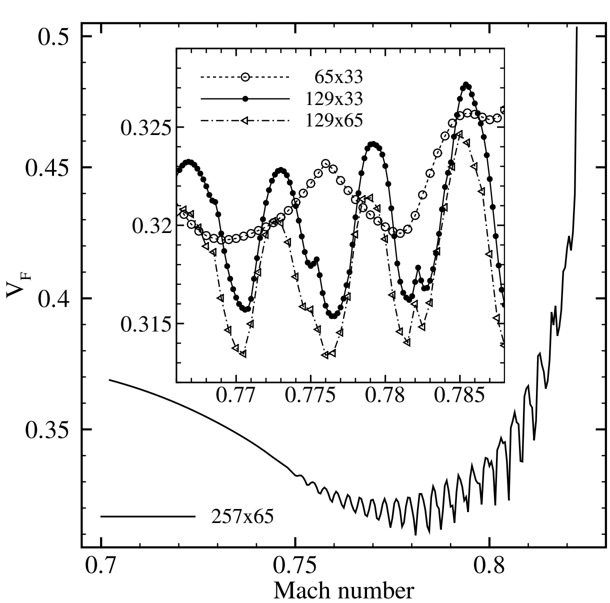

Fig. 2 Transonic instability boundary of NACA 0012 aerofoil configuration at zero angle of attack. The inlay shows the influence of grid refinement.

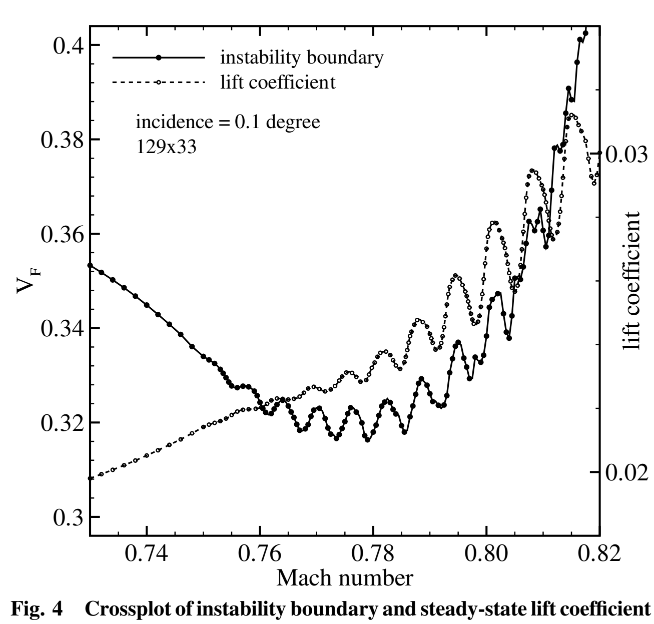

Fig. 4 Crossplot of instability boundary and steady-state lift coefficient for NACA 0012 aerofoil at a small angle of attack.

flow with moving shock wave at fixed Mach number, the discrete shock resolution was found to be responsible for an oscillatory effect [15]. The shock capturing schemes giving the crispest shock resolution were shown to result in the highest discrepancies between various integrated flow quantities and their exact analytical solutions. Increasing the level of artificial viscosity at the expense of losing the shock resolution reduced the associated error (compare section 4.1 of [15]).

In [16] an oscillatory behavior was observed in inviscid steady-state simulations employing a discrete adjoint method. Therein, for instance, lift slopes of a NACA 0012 aerofoil obtained from linear and adjoint codes with base flows being the nonlinear steady states over a limited range of angles of attack, and two fixed Mach numbers were compared with the slopes obtained by finite differencing of the nonlinear lift coefficients. For a subsonic freestream Mach number the differences between the linear/adjoint results and the nonlinear data were negligible. For the transonic freestream Mach number on the other hand, the nonlinear lift coefficient showed a lack of smoothness with varying angle of attack. A second example of a transonic diverging duct flow illustrated a periodic behavior of the integrated pressure with varying exit pressure. The period of the repeating pattern was determined by the grid spacing.

The appearance of nonsmooth transonic results is related to the discrete displacement of the formed shock wave under variation of a particular system parameter. The system parameter under variation in the current study is the freestream Mach number. Here in the eigenvalue-based approach, the derivatives, that is, the elements of the system Jacobian matrix, oscillate with the changing freestream Mach number. The oscillations in the instability boundary are due to changes in derivatives produced by discrete shock motions.

IV. Conclusions

An eigenvalue-based approach to predict aeroelastic instabilities in transonic flow was examined revealing an oscillatory behavior in a highly resolved (in terms of Mach number increment) instability boundary. Unsteady, time-accurate results depending on the initial disturbance matched the oscillatory prediction of the eigenvalue-based approach. The observed phenomenon was related to the discrete shock motion between grid points. This behavior is consistent with previous studies of the influence of discrete shock resolution in inviscid flow models.

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